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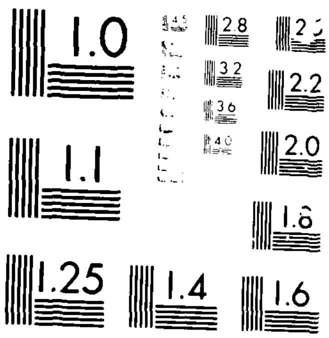

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This report describes the research accomplishments under grant 80-0196. Applications are described of various mathematical techniques to problems of regulation and control of nonlinear sampled-data systems and of systems over rings, including delay-differential systems and families of linear systems. An extensive bibliography of papers published is included.

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FINAL TECHNICAL REPORT - AFOSR 80-0196

The work of the PI during the years covered by the grant has always emphasized *discrete-time nonlinear* system theory as well as algebraic methods in the analysis of *generalized* classes of *linear* systems.

A *system*, as understood in this general area of research, is a precise mathematical object which models a controlled and observed dynamic process. The central concept in this model is the *state* of the system, represented by a suitable set of parameters. For instance, the state of a rigid body is specified by the position and (linear) momentum of its center of mass (6 parameters), together with its attitude and angular momentum (6 more parameters). An important observation in this example is that the natural way to represent the attitude is by means of three positively oriented mutually perpendicular vectors -in technical jargon, an element of the Special Orthogonal Group. This is an instance of a more general situation: states take values on suitable mathematical spaces -differentiable manifolds, algebraic varieties, or just linear spaces,- whose structure and properties reflect the various constraints on parameters. For a related example, a robotic manipulator is modeled in an analogous fashion, with more complicated constraints due to the interactions between links and to obstacles in the workspace.

System dynamics are usually represented by difference or by differential equations. These equations contain forcing terms, *inputs* or *controls*, with a great number of problems in system theory having to do with the appropiate choice of controls in order to achieve desired objectives: bring a rigid body -e.g. a satellite- to a proper position and attitude through the firing of appropiate jets, control a manipulator by applying torques at each joint, and so on. An *observation* or *measurement* function is often explicitely associated to the system model, representing the information pattern available to the controller.

Most physical dynamics are modeled by differential equations, so system theory has traditionally concentrated on such *continuous-time* systems. In modern digital control, however, physical systems interact with discrete devices. It then becomes natural to focous attention on behavior at appropiate sampling times, and this gives rise to *discrete-time* systems, modeled by difference equations. When only small perturbations from equilibrium are involved, -stabilizing a satellite moving very slowly, for instance,- the study of "linear" systems, i.e., the theory based on first-order approximations of dynamics and observations, has proved extraordinarly successful -viz. the "Kalman filter" and other widely applied results. In that context, very few conceptual differences arise between the continuous and discrete cases. (Technically, this is due to the many shared linear-algebraic properties of differential and difference operators.)

When dealing with more global phenomena, like moving a robot arm from a given configuration into a totally different one, one cannot rely solely on linear methods, and the development of a truly nonlinear theory becomes essential. For various technical reasons, the studies of continuous and discrete nonlinear systems diverge considerably. In contrast to the linear case, then, the development of a methodology for sampled nonlinear systems does not follow from that for the continuous case. Much of the PI's work during the past few years hs concentrated on the development of such a methodology. Among past accomplishments one may mention: the development of a synthesis theory for input/output behaviors with polynomial nonlinearities, (which resulted in a Springer Lecture Notes volume,) more recent work under this grant on discontinuous (*piecewise linear*) control, general results on regulation with partial information, and the construction of energy-like ("Lyapunov") functions to characterize controllability. These results have given rise to various publications during the past few years, -see* 3,17,5,6,10-12,16,26 - as well as to further research by other authors. On the practical side, a large current research effort on

*Numbers refer to enclosed list of publications under this grant

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the modeling of nuclear reactors, carried out by engineers at Electricite de France, as well as a recent Chemical Engineering dissertation at Princeton, also concerned with reactor modeling, are both based largely on the PI's results on nonlinear discrete time systems. Present research in this general area has started to shift into questions of computation: the hope being that eventually an approach based on piecewise linear approximations will be feasible for computer-aided control design: the papers [4] and [25] point in this direction. The paper [25], for instance, provides a precise computational complexity characterization of the constructions in [4].

During the past three years, we have started research into the problem of *sampling* a continuous system, namely, to decide what objectives can be achieved, at least in principle, with digital control, if they are known to be obtainable with analogue controllers. It is "folk knowledge", for instance, that certain robotic manipulators "should" be sampled at no less than 60 Hz or so, in order to avoid interactions with natural modes of the system. But this intuition is based on a linearized analysis, and may well be too conservative -or even totally inappropiate- for a particular (global) control task. We have obtained a large number of results insuring that certain properties like controllablity are preserved under sampling under high enough rates, provided that the parameter space satisfies certain easily checked topological constraints (technically, a condition on its first homotopy group). Both robotics and satellite control problems can be seen to satisfy the above topological hypothesis, for example, but something like the "60 Hz" rule mentioned earlier cannot as yet be formulated as a consequence of the general results. The PI's current research in this area is in the direction of more explicit formulas for the actual sampling frequencies needed. Mathematically, the tools needed have been borrowed mainly from differential and algebraic geometry, Lie Groups (rotating rigid body examples), elementary logic (piecewise linear systems), and optimization theory. See [13,15,20].

One success in the direction of explicit characterizations of sampling frequencies has been the following. For linear systems, the classical result is due to Kalman, Bertram, Ho, and

Narendra, who established that controllability when sampling at frequency $1/\delta$ is preserved if $\delta(\lambda-\mu)$ is not of the form $2k\pi$ for any pair of distinct eigenvalues of the A matrix. In research during this last year (see [27,33]), we have found a general result which in particular implies, for the large class of *bi*linear systems, an analogous property: one now needs that $\delta(\lambda-\lambda'-\mu-\mu')$ not be equal to $2k\pi$, k non zero, for any four eigenvalues of the autonomous dynamics matrix. Thus in the bilinear case, one must sample at 4 times (rather than twice) the natural frequencies of the system. The result is obtained by inducing a linear system on the adjoint representation of a certain Lie algebra associated to the given system.

Other recent discrete-time nonlinear research supported by this grant is that of finite computations in the field of stochastic estimation. Given a time series, it is often possible to compute ***sufficient statistics*** of the associated process, estimators which serve to predict future samples of the series. This is closely related to issues of nonlinear filtering -estimating the state of a partially observed process,- and identification -obtaining a model to account for the physical system generating the observed process. The question of computing and dynamically updating sufficient statistics with finite resources had received almost no attention in the literature, and turns out to be technically related to the previous work on discrete systems by the PI. Since late 1982 we have collaborated in this area with Bradley Dickinson from the EE dept. at Princeton. As a result, we obtained characterizations of the complexity of updating equations for some important classes of processes; see [18,30].

Other areas of recent nonlinear research have dealt with questions of nonlinear observability, with feedback transformations that simplify models of dynamical systems, and with the development of small-control results. These have resulted in a number of recent papers; see [14,24,29]. Another such area, still in progress ([34]), is related to the topic of linearization under dynamic compensation. We have proposed a weaker notion of linearizability (related to work of Rugh and others), and proved that, under very mild technical assumptions, a nonlinear controllable system admits a precompensator with the following property: along control trajectories joining pairs of states, the composite system is up to first order a parallel connection of integrators.

Another major area has been the study of *generalized linear* systems. These are classes of models for which the basic dynamics are still *linear*, but which do not fit into standard models by difference or differential equations. One of these is that of *delay-differential* systems, which arise when transmission delays cannot be ignored. Another class of models concerns *families of systems*, which serve to study simultaneously a set of linear system parametrized in some way; typically, one deals with the set of first-order approximat[illegible] a nonlinear system around different operating points, or with systems with paran[illegible] which are unknown to the syetem designer. Past results published under this grant [illegible] major influence on a major theoretical advance in 1983/84, the proof by a group at the University of Florida (E.Kamen at al.) of the sufficiency fo finite dimensional controllers for stabilizing delay systems, a result that is bound to have major practical implications, given the widespread use of such systems. (The methods are based in part on the inversion of a properly chosen polynomial matrix, which originated with the work of the PI with M.Hautus in (2).) Related results by the PI on families of systems have been incorporated into an approach to adaptive control recently introduced by E.Emre. More recent work by the PI dealt with the stabilization of families of linear systems using polynomially parametrized feedback; these methods can be useful in reducing the amount of on-line computation needed in the above adaptive control scheme. It should be pointed out that adaptive control techniques are currently being explored in a variety of industrial applications; it is very likely that some of this algebraic work will find its way into such practical applications in the near future. See (1,2,3,7,8,19,23) for publications in this area.

More recently, we have completed a couple of papers on pole-shifting problems and on the existence of stabilizers for parametrized systems. One of the main results on families of systems (29) shows that a rationally parametrized set of sytems can be stabilized by a dynamic compensator whose entries are also rationally parametrized, both in the discrete and continuous cases. The only hypothesis needed is that of pointwise stabilizability. This improves considerably on results by the PI and many others, which typically require controllability. Another result (28), obtained jointly with M. Hautus, is that periodically parametrized feedback is always possible for similarly parametrized families. The paper 29 also provides a tutorial introduction to the topic of control of families of systems.

Finally, we have recently (31) obtained various results on the optimal control of robotic manipulators. This research, coauthored by H.Sussmann, applies modern algebraic techniques to the characterization of singular and optimal trajectories for two-link planar manipulators. All previous results in this area relied on numerical methods. We expect our results to provide a better conceptual understanding and to lead eventually to more efficient numerical methods. Similarly, recent work (also with H.Sussmann) on image recognition (32) has been based on recent research on stochastic system theory.

Many of the system over rings results, as well as those on nonlinear systems, are basically constructive, and we have worked in the related area of designing algorithms and [illegible] the complexity of the corresponding constructions. The work 25 mentioned earlier [illegible] the complexity type. The robotics work 31 relies totally on calculations using symbolic manipulation systems.

Applications, like those mentioned at various points above, are encouraged by the PI through publications as well as discussions and presentations at conferences, universities, and industrial laboratories. For instance, the French (and Princeton) work on reactor modeling arose in this way. The research by the PI, however, is and will continue to be directed towards a basic mathematical understanding of the system-theoretic questions involved.

Papers submitted under previous AFOSR Grant (79 80), but referred to in this report.

1. "On generalized inverses of polynomial and other matrices". *IEEE Trans. Autom. Contr.*, *AC-25*(1980): 514-517.

2. (With M.L.J.Hautus,)"An approach to detectability and observers", in *AMS-SIAM Symposia in Applied Math., Harvard, 1979* (Byrnes,C. and Martin,C., eds.): 99-136. AMS-SIAM Publications, 1980.

3. (With R.Bumby, H.Sussmann, and W.Vasconcelos,) "Remarks on the pole-shifting problem over rings", *J. Pure Appl. Algebra*, **20**(1981): 113-127

Papers submitted and published under grant.

4. "Remarks on piecewise-linear algebra", *Pacific J.Math.*, **98**(1982): 183-201.

5. (With H.Sussmann) "Remarks on continuous feedback", *Proc. IEEE Conf. Dec. and Control, Albuquerque, Dec.1980.*

6. "Nonlinear regulation: The piecewise linear approach", *IEEE Trans.Autom.Control* **AC-26**(1981): 346-358.

7. "Linear systems over commutative rings: a (partial) updated survey", *Proc. IFAC VIII Triennial World Congress, Kyoto, Aug.1981.*

8. (With P.P.Khargonekar) "On the relation between stable matrix fraction decompositions and regulable realizations of systems over rings", *IEEE Trans.Autom. Control*, **27**(1982): 627-638.

9. "A Lyapunov-like characterization of asymptotic controllability", *SIAM J. Control and Opt.*, **21**(1983):462-471.

10. "A characterization of asymptotic controllability", in *Dynamical Systems II* (A.Bednarek and L.Cesari, eds.), Academic Press, NY, 1981.

11. "Abstract regulation of nonlinear systems: stabilization", *Proc. Symp.on Feedback and Synthesis on Linear and Nonlinear Systems, Bielefeld and Rome, June July 1981*; appeared in Springer Lecture Notes in Control and Information Sciences, Springer, 1982.

12. "Abstract regulation of nonlinear systems: stabilization, Part II", *Proc. Conference Info. Sci. and Systems, Princeton, Mar. 1982*, pp.431-435.

13. (With H. Sussmann) "Accessibility under sampling", *Proc. IEEE Conf. Dec. and Control, Orlando, Dec. 1982.*

14. "An algebraic approach to bounded controllability of linear systems", *Int. J. Control* **39**(1984): 181-184.

15. "On the preservation of certain controllability properties under sampling", in *Developpement et Utilization d'Outiles et Modeles Mathematiques en Automatique, Analise de Systemes et Traitement de Signal, Coll. CNRS, RCP 567*, Belle-Ile, France, 1983, pp.623-637.

16. "Conditions for abstract nonlinear regulation", *Information and Control*, **51**(1982):105-127.

17. "Reachability, observability, and realization of a class of discrete-time nonlinear systems," in *Encycl. of Systems and Control*, Pergamon Press, 1984.

18. (With B.Dickinson and C.A.Schwartz) "Characterizing innovations representations for discrete-time random processes", *Stochastics*, **11**(1984): 159-172.

19. (With R.Bumby) "Stabilization of polynomially parametrizad families of linear systems. The single input case," *Systems and Control Letters*, **3**(1983): 251-254.

20. "An approximation theorem in nonlinear sampling," in *Mathematical Theory of Networks and Systems*, (P.A.Fuhrmann, ed.), Springer, Berlin, 1984, pp.806-812.

21. (By M. Fliess, post-doctoral visitor, 1982-83) "Lie Brackets and nonlinear optimal feedback regulation," in *Proceedings of the 9th. World IFAC Congress*, Budapest, 1984.

22. (By M. Fliess, see 21 above) "On the inversion of nonlinear multivariable systems," in *Mathematical Theory of Networks and Systems*, (P.A.Fuhrmann, ed.), Springer, Berlin, 1984, pp.323-330.

23. "Parametric stabilization is easy," *System and Control Letters* **4**(1984): 181-188. to appear.

24. "A concept of local observability," *System and Control Letters* **5**(1985): 41-47.

25. "Real addition and the polynomial hierarchy," *Inform.Proc. Letters* **20**(1985): 115-120.

26. "Remarks on input output linearization," *Proc. IEEE Conf. Dec. and Control*, Las Vegas, Dec. 1984, pp. 409-412.

27. "Further results on accessibility under sampling," *Proc.Conf. Info. Sci. and Systems*, Johns Hopkins University, March 1985.

Papers accepted for publication but not yet appeared.

28. (With M.L.J.Hautus) "New results on pole-shifting for parametrized families of systems", *J.Pure Applied Algebra*: to appear.

29. "An introduction to the stabilization problem for parametrized families of systems," to appear in *Contemporary Mathematics: Linear Algebra and its Applications to System Theory* (B.Datta, ed.), AMS-SIAM Publications.

30. (With B. Dickinson) "Dynamic realizations of sufficient sequences", *IEEE Trans.Information Theory*: to appear.

31. (With H. Sussmann) "Remarks on the time-optimal control of two-link manipulators,"

IEEE Conf. Dec. and Control, 1985.

32. (With H. Sussmann) "Image restoration and the annealing algorithm". IEEE Conf. Dec. and Control, 1985.

Papers in preparation.

33. "Orbit theorems and sampling", final version in preparation for Proc. Conf. Nonlinear Control and Applications, Paris, June 1985.

34. "Linearized regulation", to be submitted.

Lectures and Seminars during grant period:

Texas Tech. (Dec. 80), *Berkeley* (Feb. 81), *NASA -Ames Research Center* (Feb. 81), *USC* (Feb. 81), *UCLA* (Feb. 81), *Florida* (Mar. 81), *Swiss Fed. Inst. of Technology* (July 81), *Inst.of Math. of the Polish Acad.of Sciences* (Aug. 81), *U. Paris* (Sept. 82), *Florida* (Dec. 82), *Florida Atlantic* (Dec.82, May 82), *Lockheed Electronics* (Jan. 83), *Weizmann Inst. of Science* (June 83), *Univ. of Maryland* (Mar. 84).

Conference talks during grant period:

Conf.on Algebraic System Theory, Harvard (I), Cambridge, 1979; *Workshop Math. Theory Networks and Systems* (I), Virginia Beach, 1980; *IEEE-CAS Workshop on Nonlinear Networks and Systems* (short paper), Houston, 1980; *IEEE Conf. Dec. and Control*, Albuquerque, 1980; *Int. Conf. Dynamical Systems*, Gainesville, 1981; *Worshop on Feedback and Synthesis of Linear and Nonlinear Systems* (I), Bielefeld (W.Germany) and Rome, 1981; *IFAC Triennial Congress, Special Session on Algebraic System Theory* (I) (paper not delivered in person), Kyoto, 1981; *Princeton Conf. Info. Sc. and Systems*, Princeton, 1982; *Workshop on Math. Methods for Control, Systems Analysis, and Signal Proc.*, Belle Ille, France, 1982; *IEEE Conf. Dec. and Control* (two papers, one invited), Orlando, 1982; *Johns Hopkins Conf. Info. Sci. and Systems*, 1983; *Int. Conf. Math. Theory Networks and Systems*, Beer Sheva, 1983; *US-Japan Conference on System Theory* (I), Gainesville, 1983; *IEEE Conf. Dec. and Control* (I, 2 papers), San Antonio, 1983; *AMS Conf. on Linear Algebra and its Applications to System Theory, Orono, 1984*; *IEEE Conf. Dec. and Control*, Las Vegas, 1984; *Johns Hopkins Conf.Info. Sci. and Systems*, Baltimore, 1985; *Conf. Nonlinear Control and Applications*, Paris, June 1985.

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